A Cost Efficient Online Algorithm for Automotive Idling Reduction

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ABSTRACT

Idling, or running the engine when the vehicle is not moving, accounts for 13% - 23% of vehicle driving time and costs billions of gallons of fuel each year. In this paper, we consider the problem of idling reduction under the uncertainty of vehicle stop time. We abstract it as a classic ski rental problem, and propose a constrained version with two statistics μ_{B^-} and q_{B^+}, the expectation of short stops' lengths and the probability of long stops. We develop an online algorithm that combines the best of the well-known deterministic and randomized schemes to minimize the worst case competitive ratio. We demonstrate the robustness of the algorithm in terms of both worst case guarantee and average case performance using simulation and real-world driving data.

1. INTRODUCTION

Fuel economy has become a major concern in vehicle designs, due to its significant environmental impact and the foreseeable shortage of fossil oil. There is an enormous amount of efforts in place to reduce the vehicle fuel consumption and emission (see e.g. [8]). This greatly motivates the development and commercialization of electric vehicles, hybrid electric vehicles, and other energy efficient vehicles.

In this paper, we consider the problem of **reducing the cost associated with vehicle idling**. An idling vehicle runs its engine when it is not moving, which causes unnecessary waste of fuel. The average amount of idling has been measured at 13% to 23% of the total vehicle operating time, according to surveys conducted in North America and Europe [4]. In US alone, idling vehicles uses more than 6 billion gallons of fuel at a cost of more than $20 billion each year [1]. These (possibly astonishing) facts have triggered significant legislation efforts against unnecessary long idling. For example, Toronto City Council at its meeting on July 8, 2010, made changes to the Idling Control By-Law, to impose an idling limit of 1 minute [7]. Similar rules and laws can be found throughout US [3] and Europe [6].

DAC '14, June 01 - 05 2014, San Francisco, CA, USA

In order to reduce the costs associated with idling time (including fuel and emissions), the driver may manually turn off the engine, when he/she expects to experience a long stop. Alternatively, Stop-Start Systems (SSS) have been proposed to automatically perform the task. Such a system is a key building block in hybrid electric vehicles (HEV), but it can also be added as a new feature to conventional vehicles (those equipped with an internal combustion engine only). In the later case, they are typically referred to as Stop-Start Vehicles (SSV) or micro-hybrid vehicles. SSV would turn off the engine immediately when the car stops, and restart the engine when the driver pushes the gas pedal to go forward. Other functions like accessories and lighting are powered by an electrical source other than the vehicle's alternator. In HEV, the strategy can be more complicated, and is out of the scope of this work.

As in the case of idling, restarting the engine also comes with a cost. It is estimated that the fuel consumption for restarting the engine once is equivalent to keeping the engine idling for 10 seconds [4, 2]. Considering other cost associated to engine wear and exhaust gas emission, this number goes up to 28 seconds for SSV or 47 seconds for those without SSS (see Appendix C for details). Thus, it is not necessarily the best strategy to turn off the engine immediately. Considering the cost of fuel consumption alone, it is better to keep the engine running if the vehicle is known to be at rest for less than 10 seconds.

However, the vehicle stop time is unknown or even hard to estimate in many situations, such as at traffic lights or in heavy traffic. Thus, SSV have to make *online decisions*, i.e., without the a-prior knowledge of the vehicle stop time. In this paper, we consider the problem of finding the best online strategy for the start-stop systems. It can also be provided as a driving tip to drivers of vehicles without stop-start systems. In particular, we claim the following contributions:

– We consider the costs of fuel consumption, emission, and engine wear associated with idling and restart. We abstract the problem as a classic ski rental problem, where a break-even value characterizes the trade-off between keeping the vehicle idle and restarting the engine. Thus, existing solutions can be incorporated.

– We observe the characteristic of the optimal offline algorithm, and propose a constrained ski rental problem by introducing two new statistics μ_{B^-} and q_{B^+}, where μ_{B^-} is the expected length of short stops, and q_{B^+} is the probability of long stops. We derive an online algorithm for the constrained ski rental problem, which gives the smallest worst

case expected competitive ratio under any traffic conditions.
– We use real-world data and simulation to test the performance of the online algorithm. For vehicles with or without SSS, the proposed strategy exhibits robust behavior in different traffic conditions. For 1182 vehicles with real driving data, it performs the best in 1169 vehicles if they are SSV, and in 977 vehicles without SSS. At the same time, it achieves the smallest bound on worst case performance.

The rest of the paper is organized as follows. In Section 2, we introduce the problem of SSS online strategy and link that to the classic ski rental problem. We also review related works proposed in the context of the ski rental problem. In Section 3 we consider the constrained ski rental problem. In Section 4 we propose an online algorithm to minimize the worst case CR. In Section 5, we use real-world driving data and simulation to validate the performance of the proposed strategy. Finally, the paper is concluded in Section 6.

2. IDLING REDUCTION PROBLEM

When the car has to stop due to the traffic or the driver's needs, there are two possible actions that the driver/SSS can take, each associated with different costs as below:
– **Keeping the Vehicle Idle**, which would waste fuel to keep the engine running at a relatively low speed, and consequently with exhaust gas emissions. The associated cost is proportional to the vehicle idling time.
– **Turning off the Engine**. In this case, the engine has to restart when the driver pushes the gas pedal. Restarting the engine requires a one-time cost due to 1) fuel consumption and related emission; 2) excessive engine wear, including those to the starter and battery.

Both costs can be calculated by studying the characteristics of the vehicle and the cost to each parts (e.g., starter and battery). In the end, we can use two constant numbers, $cost_{idling/s}$ denoting the cost of idling per unit time, and $cost_{restart}$ for the one-time cost to restart the engine. The ratio between these two

$$B = \frac{cost_{restart}}{cost_{idling/s}} \tag{1}$$

denotes the amount of idling time such that the total cost for idling is equal to the cost of stoping and restarting the engine. B is called the **break-even interval**, which plays a key role in the algorithm design.

During the vehicle stop, decision has to be made whether to continue waiting (and keeping the engine idle) or turn off and restart when the driver intends to move forward. If the vehicle stop time y is known in advance, it is easy to figure out the optimal strategy as follows: if y is less than B (informally, the stop is **"short"**), then it is better to keep the engine idle; otherwise (informally, in case the stop is **"long"**), the driver/SSS should turn off the engine immediately and restart later.

However, the vehicle stop time is naturally random, and in many situations, such as stops at light or in heavy traffic, it is unknown. The decision has to be made without having the input y, or in an **online** fashion. In contrast, the optimal strategy with the knowledge of y is called the **offline algorithm**. The problem of designing online algorithm to choose between continuing idling (and paying a repeating cost) or paying a one-time restart cost is exactly the topic of the classic **ski rental problem** [11]. In the ski rental problem [11], suppose a skier has to pay $1 for renting skis for one day or pay $B to buy his own. He/she cannot predict until which day he/she is still able to ski due to weather condition. Every day when he/she goes skiing, an online decision can be made on whether to rent or buy.

2.1 Competitive Analysis

Competitive analysis is a common way to evaluate online algorithms, which compares the cost incurred by the evaluated strategy with the optimal offline algorithm. For a stop with length y, we denote the offline cost by $cost_{offline}(y)$, which can be calculated as

$$cost_{offline}(y) = \begin{cases} y & 0 \le y < B \\ B & y \ge B \end{cases} \tag{2}$$

The online algorithm (deterministically or randomly) selects the amount of idling time x. We denote the cost of the online algorithm for a selected x and a given y as $cost_{online}(x, y)$. Since the vehicle will wait until x, if $y < x$, the cost is y; otherwise, the cost is the amount of idle time plus the one time restart cost.

$$cost_{online}(x, y) = \begin{cases} y & 0 \le y < x \\ x + B & y \ge x \end{cases} \tag{3}$$

The **competitive ratio** $cr(x, y)$ for a given pair of x and y is defined as the ratio between the costs of the online and offline algorithms:

$$cr(x, y) = \frac{cost_{online}(x, y)}{cost_{offline}(y)} \tag{4}$$

The **expected competitive ratio**, denoted as **CR**, is defined as the ratio between the expected cost of an online algorithm and that of the offline algorithm [12]:

$$CR = \frac{\underset{y}{\mathbb{E}}[\underset{x}{\mathbb{E}}[cost_{online}(x, y)]]}{\underset{y}{\mathbb{E}}[cost_{offline}(y)]} \tag{5}$$

Our objective is to select the strategy of idling time x such that the **worst case CR** ($\max_y CR$) is minimized.

2.2 Existing Solutions

For SSV, one strategy commonly used in the design[1] is that the engine would be turned off immediately when the car stops. This strategy (with the short name **TOI**) has a fixed cost of B for any stop length y. For vehicles without SSS, the drivers may be reluctantly to turn off the engine because of the concerns on the engine wear or other needs. This behavior (with the short name **NEV**) would certainly incur large cost when the stop time is long. In the following, we review existing online algorithms proposed in the context of the ski rental problem.

A **deterministic** online algorithm chooses a fixed x in (3). [11] proves that among all possible deterministic algorithms, the strategy of $x = B$ gives the smallest worst case $cr(x, y)$:

$$\min_x \max_y cr(x, y) = \max_y cr(B, y) = 2 \tag{6}$$

We use **DET** to denote this online algorithm.

If we consider the metric of the worst case CR, DET is not the best strategy. [12] proposes a randomized online algorithm, which can guarantee that the worst case CR is no larger than $e/(e-1)$ for any distribution of y. This bound is also proven to be the smallest that any online algorithm can provide with no further statistical information on y. This algorithm, denoted as **N-Rand**, select the idling time x based on the probability density function $P(x)$ as follows

$$P(x) = \begin{cases} \frac{1}{B(e-1)} e^{\frac{x}{B}} & 0 \le x \le B \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

[1] see e.g., http://en.wikipedia.org/wiki/Start-stop_system

[13] proposes to include the first-moment (the average) μ or second-moment of the stop length as additional statistical information. It then derives a revised randomization algorithm to minimize the largest CR', where

$$CR' = \mathbb{E}_y \left[\frac{\mathbb{E}_x[cost_{online}(x, y)]}{cost_{offline}(y)} \right] \tag{8}$$

With the available information on μ, if $\mu \le 2\frac{e-2}{e-1}B = 0.836B$, the probability density function of x is derived as in (9); otherwise, it is the same as **N-Rand**.

$$P(x) = \begin{cases} \frac{1}{B(e-2)} \left(e^{\frac{x}{B}} - 1 \right) & 0 \le x \le B \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

The upper bound on CR' is proven to be $1 + \frac{\mu}{2B(e-2)}$. We denote this strategy as **MOM-Rand**.

Other works include [10] [14]. [10] proposes to analyze the average-case CR, but the analysis is based on the assumption that the distribution $q(y)$ of stop length y is exponential or uniform. [14] defines a variance of the classic ski rental problem, by introducing the option of leasing (partly rent, partly buy) in addition to pure rent or pure buy.

In the following, we look at additional statistical information of the stop length that would help provide better performance guarantees than the existing solutions. We use the definition of CR in (5), because of its direct relationship with the expected cost of the online algorithm.

3. CONSTRAINED SKI RENTAL PROBLEM

First-moment (the average) is widely used as characteristics of random variables. However, it may not be informative for the ski rental problems. Once the stop length y is longer than B, to what extent its length exceeds B would not affect the optimal offline decision: the engine should be turned off immediately. Similarly, the behavior of the deterministic online algorithm (DET) does not depend on the actual length y either if $y > B$: it would only wait until time B to turn off the engine. In addition, we prove that the additional information μ does not change the randomized online algorithm: with any given μ, the randomized algorithm is still the same as defined in (7), and the optimal CR remains to be $\frac{e}{e-1}$. The proof is informally described in Appendix B.

We observe that the average length for stops shorter than B is still meaningful. For the stops with length higher than B, we will use its total probability. Hence, we propose to use the knowledge of μ_{B^-} and q_{B^+} to improve the online algorithm design, which are defined as follows [2]

$$\mu_{B^-} = \int_{0^+}^{B} y q(y) dy \tag{10}$$

$$q_{B^+} = 1 - \int_{0^+}^{B} q(y) dy \tag{11}$$

Now all the possible distributions of stop length y can be described by the set $\mathcal{Q}$

$$\mathcal{Q} = \{ q(y) | q(y) \ge 0, \text{ (10) and (11) are satisfied.} \} \tag{12}$$

With these two constraints, the expected costs of the offline algorithm and **DET** are

$$\mathbb{E}_y[cost_{offline}(y)] = \mu_{B^-} + q_{B^+} B \tag{13}$$

$$\mathbb{E}_y[cost_{DET}(y)] = \mu_{B^-} + 2 q_{B^+} B \tag{14}$$

which are both constants for a given pair of μ_{B^-} and q_{B^+}. Also, an upper bound on the expected offline cost can be derived as B (since $\mu_{B^-} \le B$). This is consistent with the intuition that no online algorithm can outperform the offline algorithm, including **TOI**, whose expected cost is always B.

Our problem is to find an online algorithm that defines the probability distribution $P(x)$ of the idling time x with the given information of μ_{B^-} and q_{B^+}, such that it provides the smallest upper bound on the CR (and consequently the expected online cost). If the expected online cost with strategy $P(x)$ and stop length distribution $q(y)$ is denoted as

$$J(P, q) = \mathbb{E}_y[\mathbb{E}_x[cost_{online}(x, y)]] \tag{15}$$

the problem can be formulated as a minimax problem

$$\min_{P \in \mathcal{P}} \max_{q \in \mathcal{Q}} \; J(P, q) \tag{16}$$

where $\mathcal{P}$ defines the set of all possible $P(x)$

$$\mathcal{P} = \left\{ P(x) | P(x) \ge 0, \; \int_{0^+}^{+\infty} P(x) dx = 1 \right\} \tag{17}$$

4. PROPOSED SOLUTION

We first consider the solution format. Similar to the case of randomized algorithm (N-Rand) [12], it can be prove that $\forall x > B$, $P(x) = 0$ (see Appendix A). In other words, the optimal online strategy only selects idling time x no larger than B.

Next, we observe that N-Rand has a continuous pdf for $x \in [0, B]$. The deterministic online algorithm (DET) exhibits the same optimal behavior as the offline algorithm when the stop length y is less than B. On the other hand, the solution of turning off immediately (TOI) follows the online strategy when $y > B$. Both DET and TOI can be regarded as a discrete probability distribution, represented with dirac function. Thus, we propose a generic solution format for the designer's strategy $P(x)$, to include the discrete and continuous distributions simultaneously:

$$P(x) = p(x) + \alpha\delta(x - \varepsilon) + \beta\delta(x - B) + \gamma\delta(x - b) \tag{18}$$

where $p(x)$ is a continuous pdf function, $\delta(x)$ is the Dirac delta function, and ε is an arbitrarily small positive number (to represent the algorithm TOI). In Equation (18), there are three components of discrete distributions at ε, B, and b, with a probability mass function of α, β, and γ respectively. The one at b ($0 < b < B$) is used to represent **b-DET**. The only difference between **b-DET** and **DET** is that **b-DET** would idle until b instead of B. We now use the following steps to solve the constrained ski rental problem as in (16).

First, we assume α, β, and γ are constants, and solve (16):

– The problem (16) (constrained by (10) and (11)) is transformed to an unconstrained one using the standard Augmented Lagrangian method, as in Section 4.1.

– A set of relationship between $q(y)$ and $P(x)$ is introduced to offset the variation in $q(y)$. The problem now is converted to a linear programming (LP) problem with an objective independent of $q(y)$, as in Section 4.2.

– In Section 4.3, we obtain and solve an ordinary differential equation for the continuous pdf $p(x)$, and find the Lagrangian coefficients as functions of α, β, and γ.

With the derived Lagrangian coefficients, we can transform the problem (16) into an LP with variables α, β, and γ. Their values can be solved with standard techniques in linear programming, as in Section 4.4.

4.1 The Augmented Lagrangian

[2] Mathematically speaking, the expectation of short stops should be $\frac{\mu_{B^-}}{1-q_{B^+}}$. We use our definition for convenience.

We denote the expected online cost for a given $y \le B$ as

$$C(P(x), y) = \int_{0^+}^{y} (x+B)P(x)dx + \int_{y}^{B} yP(x)dx \quad (19)$$

and the one for $y > B$ as

$$C'(P(x), y) = \int_{0^+}^{B} (x+B)P(x)dx \quad (20)$$

The expected online cost $J(P,q)$ can be represented as

$$\begin{aligned} J(P,q) &= \int_{0^+}^{+\infty} \mathbb{E}_x[cost_{online}(x,y)]q(y)dy \\ &= \int_{0^+}^{B} C(P(x),y)q(y)dy + \int_{B}^{+\infty} C'(P(x),y)q(y)dy \end{aligned} \quad (21)$$

In order to incorporate the constraints (10) and (11), we use Lagrange Multipliers λ_1 and λ_2 to associate the constraints with the objective function.

$$\begin{aligned} L(P,q,\lambda_1,\lambda_2) = \ & J(P,q) + \lambda_1 \left(-\int_{0^+}^{B} q(y)dy + 1 - q_{B^+}\right) \\ & + \lambda_2 \left(-\int_{0^+}^{B} yq(y)dy + \mu_{B^-}\right) \end{aligned} \quad (22)$$

Due to the linearity of $J(P,q)$ on q, strong duality holds. Now the original minimax problem (16) can be reformulated as an unconstrained one, with its objective defined below

$$\min_{P\in\mathcal{P}, q\in\mathcal{Q}} L(P,q,\lambda_1,\lambda_2) \quad (23)$$

4.2 Constraints on $P(x)$ and $q(y)$

The Lagrangian in (22) can be partitioned into two parts

$$L(P,q,\lambda_1,\lambda_2) = Obj + Con \quad (24)$$

where

$$\begin{aligned} Obj(q,\lambda_1,\lambda_2) = \ & q_{B^+} \int_{0^+}^{B} (x+B)\, p(x)dx + \alpha q_{B^+} B + 2\beta q_{B^+} B \\ & + \gamma \left(\mu_1 + (q_2 + q_{B^+})(b+B)\right) + \lambda_1 (1 - q_{B^+}) + \lambda_2 \mu_{B^-} \end{aligned} \quad (25)$$

$$Con = C(P(x), y) - \lambda_1 - \lambda_2 y \quad (26)$$

It should be noted that μ_1 and q_2, defined in (27), are variables.

$$\mu_1 = \int_{0^+}^{b} yq(y)dy \qquad q_2 = \int_{b}^{B} q(y)dy \quad (27)$$

We use the same technique as in [13] to convert (24) into a linear programming problem. The observation is that for arbitrary distribution $q(y)$ of stop length, there is a corresponding decision distribution $P(x)$ which can offset the variation from $q(y)$. This is possible as $P(x)$ can be any valid probability distribution function. The resulted problem is

$$\min_{q}\ Obj \quad (28a)$$

$$s.t.\quad Con = C(\tilde{P}(x), y) - \lambda_1 - \lambda_2 y = 0 \quad (28b)$$

$$\int_{0^+}^{B} p(x)dx = 1 - \alpha - \beta - \gamma \quad (28c)$$

$$p(x) \ge 0 \quad (28d)$$

where $\tilde{P}(x) = P(x) - \gamma\delta(x-b)$ is introduced for convenience.

4.3 Solving $p(x)$

The LP problem (28) can be solved with similar steps as in [13]. First, to find $p(x)$, (28b) is differentiated twice to derive the following ordinary differential equation (ODE):

$$\frac{d}{dx}p(x) = \frac{1}{B}p(x) \quad (29)$$

The solution for this ODE is

$$p(x) = C_0 e^{\frac{x}{B}} \quad (30)$$

where the coefficient $C_0 = \frac{1-\alpha-\beta-\gamma}{B(e-1)}$ by considering the constraint (28c). Substituting (30) into (26), we can get the Lagrange multipliers (as functions of α, β).

$$\begin{cases} \lambda_1 = \alpha B \\ \lambda_2 = BC_0 e + \beta = (1-\alpha-\beta-\gamma)\frac{e}{e-1} + \beta \end{cases} \quad (31)$$

4.4 Solving α, β, and γ

Substituting (31) into (25), the objective Obj is now a function of α, β, and γ, as in (32).

$$\min_{\alpha,\beta,\gamma} \quad K_\alpha \alpha + K_\beta \beta + K_\gamma \gamma + \frac{e}{e-1}\left(\mu_{B^-} + q_{B^+} B\right) \quad (32)$$

where K_α, K_β, and K_γ are constants, defined as

$$\begin{aligned} K_\alpha &= -\tfrac{e}{e-1}(\mu_{B^-} + q_{B^+}B) + B \\ K_\beta &= -\tfrac{e}{e-1}(\mu_{B^-} + q_{B^+}B) + (\mu_{B^-} + 2q_{B^+}B) \\ K_\gamma &= -\tfrac{e}{e-1}(\mu_{B^-} + q_{B^+}B) + [\mu_1 + (q_2 + q_{B^+})(b+B)] \end{aligned}$$

We incorporate the constraints that $P(x)$ should be a valid probability function

$$\alpha + \beta + \gamma \le 1,\ \ \alpha \ge 0,\ \ \beta \ge 0,\ \ \gamma \ge 0 \quad (33)$$

The LP problem with the objective in (32) and constraints in (33) can be solved using standard techniques in linear programming. Simply speaking, the constraints in (33) limit that α, β, and γ are all finite. By the fundamental theorem in linear programming, the solution space of this LP problem forms a convex polytope, and the optimal solution is obtained in one of the four vertexes. The strategy and associated cost to each vertex are summarized below:

– $(\alpha,\beta,\gamma) = (0,0,0)$: the strategy is **N-Rand**, with cost $\mathbb{E}_y[cost_{N-Rand}(y)] = \frac{e}{e-1}(\mu_{B^-} + q_{B^+}B)$ [12], worst case CR $CR_{N-Rand} = \frac{e}{e-1}$;

– $(\alpha,\beta,\gamma) = (1,0,0)$: the strategy is **TOI**, with cost $\mathbb{E}_y[cost_{TOI}(y)] = B$, worst case CR $CR_{TOI} = \frac{B}{\mu_{B^-}+q_{B^+}B}$;

– $(\alpha,\beta,\gamma) = (0,1,0)$: the strategy is **DET**, with cost $\mathbb{E}_y[cost_{DET}(y)] = \mu_{B^-} + 2q_{B^+}B$, worst case CR $CR_{DET} = \frac{\mu_{B^-}+2q_{B^+}B}{\mu_{B^-}+q_{B^+}B}$;

– $(\alpha,\beta,\gamma) = (0,0,1)$: the strategy is **b-DET**, with cost and worst case CR defined in (35) and (38) respectively, if the condition (36) is satisfied; otherwise its cost is $(b+B)$.

We now detail how the expected cost of **b-DET** is calculated. Please note that $b \in [0,B]$ is a design variable that can be selected to minimize the cost of **b-DET**.

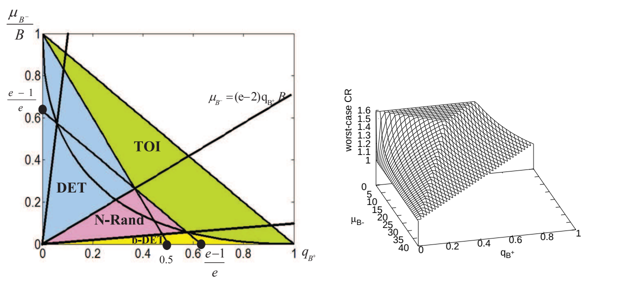
Given a pair of μ_{B^-} and q_{B^+} values, we first prove that b should select some value larger than $\frac{\mu_{B^-}}{1-q_{B^+}}$. To prove it, the stop length y can be selected to be $\frac{\mu_{B^-}}{1-q_{B^+}}$ with probability of $1-q_{B^+}$, and an arbitrary value $b' > B$ with probability of q_{B^+}. Under such a distribution of y, the expected cost of **b-DET** is $b+B$, always larger than the one ($=B$) of **TOI**. Thus **b-DET** will never be selected.

With the assumption that $b > \frac{\mu_{B^-}}{1-q_{B^+}}$, y cannot be always $\ge b$. Intuitively, any stop with length $y \ge b$ will introduce a cost of $b+B$, larger than the case $y < b$. The worst case $q(y)$ can be proven to follow the rule that all short stops have a length of either 0 or b, consequently $\mu_1 = 0$ and $q_2 = \frac{\mu_{B^-}}{b}$. The expected cost for **b-DET** is

$$\begin{aligned} \mathbb{E}_y[cost_{b-DET}(y)] &= \min_b \{\mathbb{E}_y[cost_{online}(b,y)]\} \\ &= \min_b \{(b+B)(\tfrac{\mu_{B^-}}{b} + q_{B^+})\} \end{aligned} \quad (34)$$

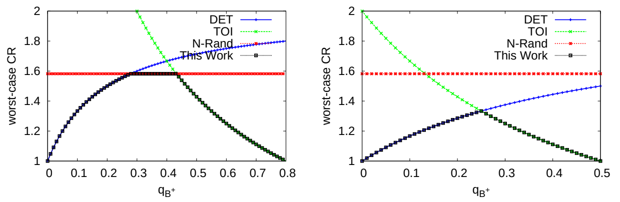
When $b = \sqrt{\frac{\mu_{B^-}B}{q_{B^+}}}$, (34) reaches its minimum value

$$\mathbb{E}_y[cost_{b-DET}(y)] = (\sqrt{\mu_{B^-}} + \sqrt{q_{B^+}B})^2 \quad (35)$$

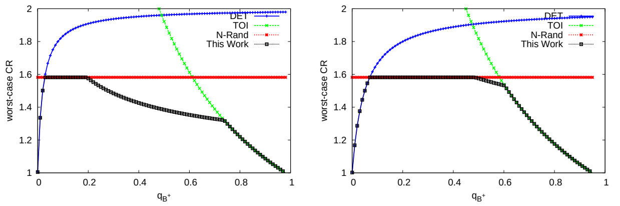


(a) Switching Rule Map (b) Worst Case CR

Figure 1: The Proposed Online Algorithm and its Worst Case CR



(a) $\mu_{B^-} = 0.2B$ (b) $\mu_{B^-} = 0.5B$



(c) $\mu_{B^-} = 0.02B$ (d) $\mu_{B^-} = 0.05B$

Figure 2: Projected view

This requires that $b = \sqrt{\frac{\mu_{B^-} B}{q_{B^+}}} > \frac{\mu_{B^-}}{1-q_{B^+}}$, or equivalently

$$\frac{\mu_{B^-}}{B} < \frac{(1-q_{B^+})^2}{q_{B^+}} \tag{36}$$

We now summarize the optimal online algorithm. In particular, it will always selects the one with the smallest expected cost among the above four strategies. For example, if the following set of inequalities are satisfied,

$$\begin{array}{l} (\sqrt{\mu_{B^-}} + \sqrt{q_{B^+}B})^2 \leq B \\ (\sqrt{\mu_{B^-}} + \sqrt{q_{B^+}B})^2 \leq \frac{e}{e-1}(\mu_{B^-} + q_{B^+}B) \\ (\sqrt{\mu_{B^-}} + \sqrt{q_{B^+}B})^2 \leq \mu_{B^-} + 2q_{B^+}B \\ \frac{\mu_{B^-}}{B} < \frac{(1-q_{B^+})^2}{q_{B^+}} \end{array} \tag{37}$$

b-DET is guaranteed to have the smallest cost among all the strategies by the first three inequalities, and the fourth inequality makes sure that there is a b that can achieve the minimum cost for **b-DET**. Hence, **b-DET** is the optimal strategy, and the worst case CR is

$$CR = CR_{b-DET} = \frac{(\sqrt{\mu_{B^-}} + \sqrt{q_{B^+}B})^2}{\mu_{B^-} + q_{B^+}B} \tag{38}$$

The solution is visualized in Figure 1. Figure 1(a) illustrates the selection of **N-Rand**, **DET**, **TOI**, and **b-DET** depending on the different values of μ_{B^-} and q_{B^+}. Figure 1(b) shows the derived worst case CR. For better comparison with these strategies, we also give two projected views in Figure 2. This figure demonstrates that our online algorithm combines the best of the well-known deterministic and randomized schemes (its worst case CR is the minimal among **N-Rand**, **DET**, **TOI**, and **b-DET**). The possible improvements brought by **b-DET** are demonstrated in Figure 2(c)-(d) (with $\mu_{B^-} = 0.02B$ and $\mu_{B^-} = 0.05B$ respectively).

5. EXPERIMENTAL RESULTS

In this section, we conduct experiments to evaluate the performance of the proposed online algorithm. We consider both SSV and the vehicles without start-stop systems. We estimate a minimum break-even interval $B = 28$ seconds for SSV, and 47 seconds otherwise (the details can be found in Appendix C). In summary, we consider both the fuel consumption and mechanical wears. Hence, our algorithm addresses not only the environmental impact of vehicle idling reduction, but also car owners' concerns on damages to car starter/battery (possible reasons why they are reluctant to shut down engines during idling).

We first use real-world driving data to demonstrate the performance of our proposed control strategy and its advantage compared with current solutions. We select data released by the National Renewable Energy Laboratory (NREL) [5] in United States. These data are collected from three areas: California, Chicago, and Atlanta, with 217, 312, and 653 vehicles, respectively. For each vehicle, the driving data were recorded for one week. Figure 3 depicts the probability distribution of the stop length for all the vehicles in these three areas. These distributions are different from the exponential distribution (as assumed in [10]) according to the Kolmogorov-Smirnov test, mostly due to their heavy tails.

We use these real-world driving data to study the CR of the proposed algorithm, and compare it with other solutions, including TOI (Turning Off Immediately), NEV (Never turning off), DET (Deterministic Algorithm) [11], N-Rand (Randomized Online Algorithm) [12], and MOM-Rand [13]. We compare both the worst case CR (the largest CR among all vehicles) and the average CR (the mean over them).

For SSV (where the break-even interval B is estimated at 28 seconds), the results are shown in the top row of Figure 4 for each of the three areas. For vehicles without SSS (where B is set to be 47 seconds), the bottom row in Figure 4 draws the comparison. From the figure, our algorithm always provides the smallest worst case CR, which is consistent with the guaranteed optimal performance. Furthermore, our algorithm also outperforms the other solutions in average CR. Among all the 1182 vehicles, our proposed algorithm achieves the best average CR in 1169 of them for SSV ($B = 28$). The mean CR of our algorithm is 1.11, 1.32, and 1.10 respectively for the three areas, lowest among all strategies. If $B = 47$ (for vehicles without SSS), our strategy achieves best performance in 977 vehicles. The mean CR is 1.35, 1.42, and 1.35 respectively, the best in each area. In summary, our algorithm not only provides the smallest upper bound on the CR, but also exhibits great performance in terms of the average CR in different areas.

Finally, we use simulation to validate the performance of the algorithm under different traffic conditions. Although the three areas have different average stop length (possibly due to different traffic conditions), their shapes of the stop length distributions are quite similar, as in Figure 3. Thus, we generate simulation driving data by following the distribution of Chicago, but scaling its mean value. We then check the worst case CR for each mean stop length.

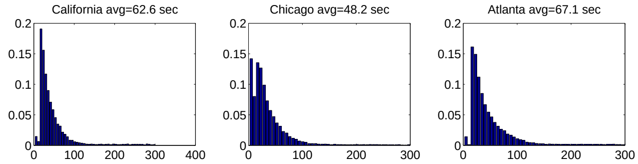
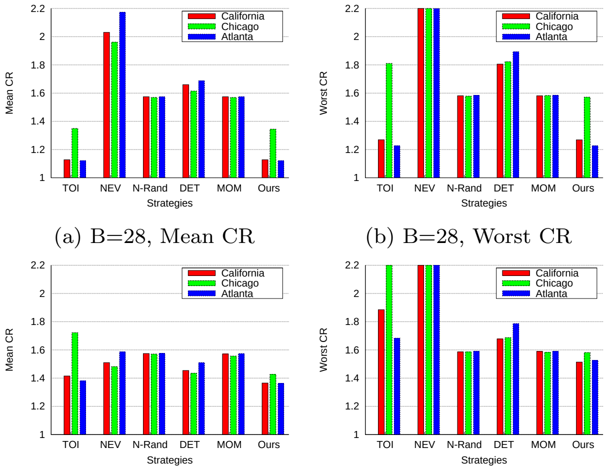


Figure 3: Distribution of Stop Length



(a) B=28, Mean CR

(b) B=28, Worst CR

(c) B=47, Mean CR

(d) B=47, Worst CR

Figure 4: Individual Vehicle Test

Figures 5 and 6 illustrate the results. It can be seen that our strategy always achieves the lowest upper bound on the CR under any traffic condition (average stop time). On the contrary, **DET** algorithm only functions well for good traffic conditions (with short average stop time), and **TOI** only works well for bad conditions (with long average stop time). The two randomized algorithms **N-Rand** and **MOM-Rand**, while being robust, is consistently outperformed by our proposed algorithm. This validates our proposal that μ_{B^-} and q_{B^+} can provide valuable information to improve the online algorithm design.

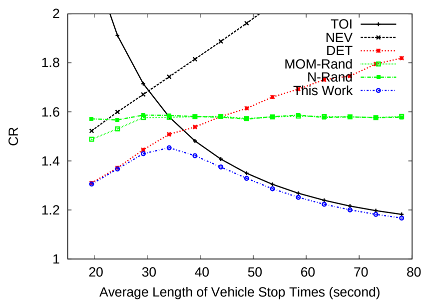


Figure 5: Worst case CR under different average stop lengths (B = 28)

6. CONCLUSIONS

In this paper, we formulate the vehicle idling reduction as the classical ski rental problem. Besides incorporating existing solutions, we propose a constrained ski rental problem with additional statistical information. We develop an online algorithm that combines the best of the deterministic and randomized schemes to minimize the worst case competitive ratio. With real-world driving data and simulation, we demonstrate that the proposed algorithm is robust and advantageous for different types of vehicles under different traffic conditions.

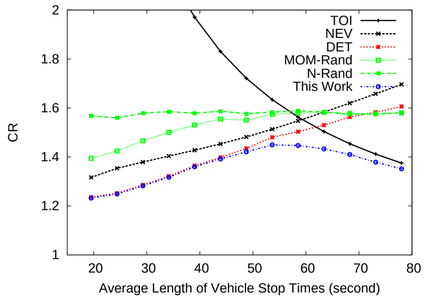


Figure 6: Worst case CR under different average stop lengths (B = 47)

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APPENDIX

A. STRATEGY SPACE RANGE

In this appendix, we prove that the strategy space of the online algorithm should be limited to $[0, B]$.

We consider the expected cost when $x = c$ where $c > B$. The expected cost with this strategy can be written as

$$\begin{aligned}
&\mathbb{E}_y[cost_{online}(c, y)] \\
&= \int_{0+}^{c} yq(y)dy + \int_{c}^{+\infty} (c + B)\, q(y)dy \\
&= \int_{0+}^{B} yq(y)dy + \int_{B}^{c} yq(y)dy \\
&+ \int_{B}^{+\infty} (c + B)\, q(y)dy - \int_{B}^{c} (c + B)\, q(y)dy \\
&= \mu_{B^-} + (c + B)\, q_{B^+} + \int_{B}^{c} yq(y)dy - \int_{B}^{c} (c + B)\, q(y)dy
\end{aligned} \tag{39}$$

For any given μ_{B^-} and q_{B^+}, the selected strategy should minimize the worst case expected CR (thus worst case expected cost as the online algorithm has a fixed cost) for any distribution $q(y)$ in $\mathcal{Q}$. However, we can construct a distribution that results in a larger cost than that of **DET**, as follows: all stops either fall in range $[0, B]$ or $[c, +\infty)$, and no stop falls in $[B, c]$. Thus, the third and fourth terms in (39) become zero, and

$$\mathbb{E}_y[cost_{online}(c, y)] = \mu_{B^-} + q_{B^+}(c+B) \geq \mu_{B^-} + 2q_{B^+}B \tag{40}$$

Now if any strategy $P(x)$ has a probability with $P(c) > 0$, we can construct another solution that add this probability to $P(B)$ with a smaller expected cost. Hence, it is safe to assume that $P(c) = 0$ for any $c > B$.

B. FIRST MOMENT CONSTRAINT

In this appendix, we prove that the added information of first moment of the stop length yield the same strategy as **N-Rand**.

The proof would be conducted by slightly modifying the Equation (7) in [13]. In order to minimize the expected online cost, instead of the metrics evaluating the performance of the online strategy, the denominator of the term $\frac{C(p(x),y)}{y}$ would be removed and the numerator $C(p(x), y)$ represents the online cost.

$$C(p(x), y) = \lambda_1 + \lambda_2 y \tag{41}$$

It should be noted that the $C(p(x), y)$ is the same as our definition of (19). By differentiating both sides of the Equation (41), we can get a first-order ODE as in (42), whose generic solution is (43). the coefficient is calculated by considering the constraint $\int_0^B p(x)dx = 1$, and the solution is updated as (44). This is the same as **N-Rand** defined in (7).

$$\frac{d}{dx}p(x) = \frac{1}{B}p(x) \tag{42}$$

$$p(x) = C_0 e^{\frac{x}{B}} \tag{43}$$

$$p(x) = \frac{1}{B(e-1)} e^{\frac{x}{B}} \tag{44}$$

Similary, we can prove that the second-moment yields the same strategy as well.

C. CALCULATION OF BREAK-EVEN INTERVAL B

In this appendix, we detail the calculation of the break-even interval B for studying the tradeoff between idling and restart. We use US dollar as the default concurrency in the following calculation.

C.1 Idling Cost

Compared with restart, the cost of idling mainly comes from the extra consumption of fuel. In engines of current generation, engine or spark plug wear/fouling caused by engine idling are too small to be measurable [4].

The fuel cost during idling is dependent on the displacement of the engine. A quantized expression can be summarized as (45) [15], where $fuel_{L/h}$ is the total fuel (in liter) consumed per hour, and D is the displacement of the engine.

$$fuel_{L/h} = 0.3644 \times D + 0.5188 \tag{45}$$

Argonne National Laboratory has taken a test on a 2011 Ford Fusion mid-sized sedan with a 2.5-L, 4-cylinder engine (175 HP) and 6-speed automatic transmission. The measured idling cost $fuel_{cc/s}$ is about 0.279cc per second. If we are interested in the monetary cost of idling $cost_{idling/s}$, it depends on price of the fuel, as in (46). If the fuel price is \$3.5 per gallon, $cost_{idling/s}$ is about 0.0258 cent/s.

$$cost_{idling/s} = fuel_{cc/s} \times \frac{p_{gallon}}{3785} \tag{46}$$

C.2 Cost of restart

For convenience, we would take the cost of idling for 1 second ($cost_{idling/s}$) as the unit to normalize all the costs associated with restart.

C.2.1 Fuel

In terms of fuel consumption, fuel incurred due to restart is equivalent to the fuel cost during 10 seconds of idling. It should be noted that this estimation was reported in several places: Chrysler Canada in 1981 and European work in 1985 [4]; NRCan's Office of Energy Efficiency on three 1999 model year vehicles [4]; and Argonne National Laboratory's work [2]. Based on this estimation applicable to vehicles in different decades, the fuel consumption of restart $B_{fuel,s/c}$ can be safely calculated as 10 seconds of idling.

C.2.2 Engine wear

Engine wear is the most important concern from drivers who may refuse to stop engine or reluctantly follow the rules. Overlooking of the engine wear from environment protection organizations along with overconcern are common. In order to convince those drivers and correct the overlooking from environment organizations, a detailed inspection is necessary. Generally, engine wear comes from the three main parts of engine: internal combustion engine (ICE) itself, starter, and battery.

$$\begin{aligned}
B_{engine,s} &= B_{mechanic,s} + B_{starter,s} + B_{battery,s} \\
B_{engine,c} &= B_{mechanic,c} + B_{starter,c} + B_{battery,c}
\end{aligned} \tag{47}$$

ICE Wear.

Despite the name "engine wear", ICE itself is the most durable among the three parts. In modern stop-start systems, ICE is modified, so that ignition is adjusted according to position of valves in order to prevent further harm on ICE. Even without SSS, there is no evidence that restarting the engine causes significant wear to ICE, and we assume that this is negligible compared to the other costs.

Starter Wear.

Compared with the mechanical parts, the starter is more vulnerable. In stop-start systems, starter is strengthened

Table 1: Stops Per Day in 3 Locations

Location	Vehicles	Mean(μ)	Std(σ)	$P\{X \leq \mu + 2\sigma\}$
Atlanta	827	10.37	8.42	0.9091
Chicago	408	12.49	9.97	0.9534
California	291	9.37	7.68	0.9553

in order to deal with more frequent stop/start operations, while conventional vehicles may suffer from that. In the following, we discuss these two cases separately.

In SSV, the starter is usually strengthened. It is reported that SSS can allow a total of 1.2 million starts [3], typically enough for a cars' lifetime. Due to the durability of SSV's starter, we estimate $B_{starter,s}$ as 0.

For conventional vehicles, starter is much more vulnerable. We use the amortized replacement cost of the starter to estimate the cost per start. We refer to the relationship between starts per day and the vehicle service life as reported in [4]. The replacement cost of a starter ranges from \$55 to \$400, depending on many factors, e.g. the length of warranty, make, model and engine size. Also, the labor cost of replacing the starter is significant, ranging from \$115 to \$225. An average cost per start $cost_{starter,c}$ can be calculated by dividing the costs of replacement and labor by durability of the starter (between 20,000 and 40,000 starts/replacement). $cost_{starter,c}$ is reported as 0.5 to 4 cents per start [4]. If the idling cost $cost_{idling/s}$ is 0.0258 cent/s, $B_{starter,c}$ ranges from 19.38 to 155.04 seconds.

Battery.

The calculation of the restart cost associated to battery is more difficult to calculate, because of the uncertainty on the number of charging/discharging times (called cyclic endurance) during a battery's lifetime. Cyclic endurance depends on the depth of discharging and the pattern of charging/discharging cycles. For example, a battery with 1.75% depth of discharge could serve for 13250 cycles before failure. When the depth of discharge increases to 31%, the number of cycles decreases to 250 [4].

Batteries for stop-start systems are usually improved in order to meet this requirement. VARTA stop-start pro batteries could provide 3 times higher levels of cyclic endurance than conventional batteries, along with a very high deep discharge capability [4].

To estimate the cost of battery per start, we use the amortized battery cost by the possible number of stops during its warranty. The most advanced stop-start battery basically needs about \$230 (without labor cost) [5], with a warranty usually 2-4 years. According to the driving data [5], the total stops per day for three different areas (Atlanta, Chicago, and California) are listed in Table 1. We consider $\mu + 2\sigma = 32.43$ as the estimated upper bound on the number of stops per day, such that 95% of the vehicles will fall in this range.

In the end, $cost_{battery,s/c}$ is calculated between 0.4841 and 0.9713 cents, and $B_{battery,s/c}$ is at least 18.76 seconds. It should be noted that this is possibly the most conservative estimation for battery cost per start.

C.2.3 Exhaust Emissions

The emission of CO_2 is proportional to the fuel consumed, thus a restart emits roughly the same amount of CO_2 as idling for 10 seconds. Evaluation on the cost of emission largely depends on legislation around the world. Carbon dioxide tax is introduced in many countries now, although with a relatively small amount. Similar to anti-idling rules, carbon dioxide tax varies a lot among different locations. The carbon tax is usually imposed for each ton of carbon dioxides emitted. Many developed countries have taxed the fuel directly for many years [6]. This cost incurred by CO_2 has already been included in the calculation of $B_{fuel,s/c}$.

Other emissions, including total hydrocarbons (THC), nitrogen oxides (NOx), and carbon monoxide (CO), are more relevant with the scrubber technology. One objection to anti-idling is that these exhaust gas emissions from restarts is significantly larger than idling, due to the cooling of catalysts. According to the measurement taken by Argonne National Laboratory [2], restart would cause emission of 44 mg THC, 6 mg NOx, and 1253 mg CO, while for every second of idling, emission of THC, NOx, and CO are 0.266 mg, 0.0097 mg, and 0.108 mg respectively.

However, the actions against these exhaust emissions have limited impact on vehicles, possibly due to its significance compared to other sources of polution. Take Sweden for example, Nitrogen Oxidant would be charged by about 4.3 Euros per kilogram of NOx (or the total emission of 166,667 restarts) [7]. Such a penalty is equal to \$0.0035 cents per restart, or the cost of an idling for 0.14 seconds.

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